MAS61015 ALGEBRAIC TOPOLOGY — PROBLEM SHEET 13

Please hand in Exercises 2 and 3 by the Wednesday lecture of Week 7. I would prefer paper, but if that is not possible for some reason, then you can send me a scan by email.

Throughout this problem sheet we use the notation of Section 18 of the notes. In particular, we use the elements $\theta_n \in C_n(\Delta_n)$ and $\kappa_n \in C_{n+1}(\Delta_n)$ that are defined in that section.

Exercise 1. Example 18.5 gives the following formula:

$$\theta_2 = \langle e_{012}, e_{12}, e_2\rangle - \langle e_{012}, e_{12}, e_1\rangle - \langle e_{012}, e_{02}, e_2\rangle + \langle e_{012}, e_{02}, e_0\rangle + \langle e_{012}, e_{01}, e_1\rangle - \langle e_{012}, e_{01}, e_0\rangle.$$

We could write this with abbreviated notation as follows:

$$\theta_2 = \langle 012, 12, 2\rangle - \langle 012, 12, 1\rangle - \langle 012, 02, 2\rangle + \langle 012, 02, 0\rangle + \langle 012, 01, 1\rangle - \langle 012, 01, 0\rangle.$$

Use the same method and the same abbreviated notation to give a formula for θ_3 (which should have 24 terms).

Exercise 2. Give formulae for κ_1 (with 3 terms) and κ_2 (with 16 terms). Use abbreviated notation as in the previous exercise.

Exercise 3. We define slightly modified versions of sd and σ as follows. Define $\mathrm{sd}'_0 \colon C_0(X) \to C_0(X)$ to be the identity, and define $\sigma'_0 \colon C_0(X) \to C_1(X)$ to be zero. Define $\lambda, \rho \colon \Delta_1 \to \Delta_1$ and $\phi \colon \Delta_2 \to \Delta_1$ by

$$\lambda(t_0, t_1) = (t_0 + t_1/2, t_1/2)$$
$$\rho(t_0, t_1) = (t_0/2, t_0/2 + t_1)$$
$$\phi(t_0, t_1, t_2) = (t_0 + t_1/2, t_1/2 + t_2).$$

For $u \colon \Delta_1 \to X$ put $\mathrm{sd}'_1(u) = u \circ \lambda + u \circ \rho \in C_1(X)$ and $\sigma'_1(u) = -(u \circ \phi) \in C_2(X)$. Extend this linearly to define $\mathrm{sd}'_1 \colon C_1(X) \to C_1(X)$ and $\sigma'_1 \colon C_1(X) \to C_2(X)$.

(a) Check that $\partial(\sigma'_1(u)) + \sigma'_0(\partial(u)) = u - \mathrm{sd}'_1(u)$.

(b) What can you say about the relationship between sd'_1 and sd_1?

Note: Here X is an arbitrary space, which may not have anything to do with $\mathbb{R}^N$. Even if $X = \mathbb{R}^N$, the map $u \colon \Delta_1 \to X$ need not be linear. Thus, you should not be using ideas or notation that are only valid for linear simplices in $\mathbb{R}^N$.